The Theoretical and Empirical Structure of the G-Cubed Model*

Warwick J. McKibbin
The Australian National University and
The Brookings Institution

and

Peter J. Wilcoxen
The University of Texas at Austin and
The Brookings Institution

ABSTRACT

This paper describes the theoretical and empirical features of G-Cubed, a multi-country, multi-sector intertemporal general equilibrium model. G-Cubed combines the attractive features of macroeconometric models and computable general equilibrium models into a unified framework. It has been used to study a variety of topics including: greenhouse gas policy, trade liberalization, tax policy and macroeconomic policy. This paper is a technical description of the model's design.

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1 Introduction

G-Cubed is a multi-country, multi-sector, intertemporal general equilibrium model that has been used to study a variety of policies in the areas of environmental regulation, tax reform, monetary and fiscal policy, and international trade.[1] It is designed to bridge the gaps between three areas of research – econometric general equilibrium modeling, international trade theory, and modern macroeconomics – by incorporating the best features of each.

From the trade literature, G-Cubed takes the approach of modeling the world economy as a set of autonomous regions – eight, in this case – interacting through bilateral trade flows.[2] Following the Armington approach (Armington 1969), goods produced in different regions are treated as imperfect substitutes.[3] Unlike most trade models, however, G-Cubed distinguishes between financial and physical capital. Financial capital is perfectly mobile between sectors and from one region to another, and is driven by forward-looking investors who respond to arbitrage opportunities. Physical capital, in contrast, is perfectly immobile once it has been installed: it cannot be moved from one sector to another or from one region to another. In addition, intertemporal budget constraints are imposed on each region: all trade deficits must eventually be repaid by future trade surpluses.

[1] For example, McKibbin and Wilcoxen (1993) examines the importance of international coordination in climate change policies; Bagnoli, McKibbin and Wilcoxen (forthcoming) explores the effect of industry-level technical change on projections of future carbon emissions.

[2] Some well-known examples of other models with international trade flows include Deardorff and Stern (1985), Burniaux, *et al.* (1992) and Hertel (1997).

[3] Given the model's level of aggregation, this is more a simple acknowledgement of reality than an assumption. Even if individual products from different countries were perfect substitutes, the aggregate products appearing in the model would not be because the composition of the aggregates differs between domestic production and imports. In motor vehicles, for example, even if there were individual domestic cars for which there were identical imported products, the mix of economy cars, luxury cars, trucks and vans in the overall motor vehicle aggregate differs between domestic production and imports.

Drawing on the general equilibrium literature, G-Cubed represents each region by its own multi-sector econometric general equilibrium model.[4] Production is broken down into twelve industries and each is represented by an econometrically-estimated cost function. Unlike many general equilibrium models, however, G-Cubed draws on macroeconomic theory by representing saving and investment as the result of forward-looking intertemporal optimization. Households maximize an intertemporal utility function subject to a lifetime budget constraint, which determines the level of saving, and firms choose investment to maximize the stock market value of their equity.[5]

Finally, G-Cubed also draws on the macroeconomic literature by representing international capital flows as the result of intertemporal optimization, and by including liquidity-constrained agents, a transactions-based money demand equation and slow nominal wage adjustment. Unlike typical macro models, however, G-Cubed has substantial sector detail and is its parameters are determined by estimation rather than calibration.

This combination of features was chosen to make G-Cubed versatile. Industry detail allows the model to be used to examine environmental and tax policies which tend to have their largest direct effects on small segments of the economy. Intertemporal modeling of investment and saving allows G-Cubed to trace out the transition of the economy between the short run and the long run. Slow wage adjustment and liquidity-constrained agents improves the empirical accuracy with which the model captures the transition. Overall, the model is designed to provide

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[4] The computable general equilibrium literature is quite large. Some well-known examples of single-country models are Johansen (1960), Dixon, Parmenter, Sutton and Vincent (1982), Ballard, Fullerton, Shoven and Whalley (1985), Jorgenson and Wilcoxen (1990), and Goulder and Summers (1989). See Shoven and Whalley (1984) for a survey.

[5] G-Cubed builds on elements from through out the literature on macroeconomics. Our representation of saving and investment, in particular, descends from Abel and Blanchard (1983). Other intertemporal general equilibrium models that include some of the features in G-Cubed include Auerbach and Kotlikoff (1987), Goulder and Summers (1989), Jorgenson and Wilcoxen (1990), McKibbin and Sachs (1991), and Goulder (1992). The latter is also described in Bovenberg and Goulder (1996).

a bridge between computable general equilibrium models, international trade models and macroeconomic models by combining the best features of each approach. The cost of this versatility is that G-Cubed is a fairly large model. It has over 5,000 equations holding in each year, is typically solved for annually for 100 years in each simulation, and has over 100 intertemporal costate variables. Nonetheless, it can be solved using software developed for a personal computer.

2 The Structure of the Model

The key features of G-Cubed are summarized in Table 1. It consists of eight economic regions: the United States; Japan; Australia; the rest of the OECD; Eastern Europe and the former Soviet Union; China; oil exporting developing countries; and all other developing countries. Within each region, production is disaggregated into twelve sectors: five energy sectors (electric utilities, natural gas utilities, petroleum refining, coal mining, and crude oil and gas extraction) and seven non-energy sectors (mining, agriculture, forestry and wood products, durable goods, non-durable goods, transportation and services). This disaggregation, summarized in Table 2, enables us to capture the sector level differences in the impact of alternative environmental policies.

Each economy or region in the model consists of several economic agents: households, the government, the financial sector and the twelve production sectors listed above. We now present an overview of the theoretical structure of the model by describing the decisions facing these agents. To keep our notation as simple as possible we have not subscripted variables by country except where needed for clarity. Throughout the discussion all quantity variables will be normalized by the economy's endowment of effective labor units. Thus, the model's long run steady state will represent an economy in a balanced growth equilibrium.

2.1 Firms

We assume that each of the twelve sectors can be represented by a price-taking firm which chooses variable inputs and its level of investment in order to maximize its stock market value. Each firm's production technology is represented by a tier-structured constant elasticity of substitution (CES) function. At the top tier, output is a function of capital, labor, energy and materials:

$$(1) \qquad Q_i = A_i^O \left(\sum_{j=K,L,E,M} \left(\delta_{ij}^O\right)^{\frac{1}{\sigma_i^O}} X_{ij}^{\frac{\sigma_i^O - 1}{\sigma_i^O}} \right)^{\frac{\sigma_i^O}{\sigma_i^O - 1}}$$

where Q_i is the output of industry i, X_{ij} is industry i's use of input j, and A_i^O, δ_{ij}^O, and σ_i^O are parameters. A_i^O reflects the level of technology, σ_i^O is the elasticity of substitution, and the δ_{ij}^O parameters reflect the weights of different inputs in production; the superscript *o* indicates that the parameters apply to the top, or "output", tier. Without loss of generality, we constrain the δ's to sum to one.

At the second tier, inputs of energy and materials, X_{iE} and X_{iM}, are themselves CES aggregates of goods and services. Energy is an aggregate of goods 1 through 5 (electricity through crude oil) and materials is an aggregate of goods 7 through 12 (mining through services). The functional form used for these tiers is identical to (1) except that the parameters of the energy tier are A_i^E, δ_{ij}^E, and σ_i^E, and those of the materials tier are A_i^M, δ_{ij}^M, and σ_i^M.

The goods and services purchased by firms are, in turn, aggregates of imported and domestic commodities which are taken to be imperfect substitutes. We assume that all agents in the economy have identical preferences over foreign and domestic varieties of each commodity.

We represent these preferences by defining twelve composite commodities that are produced from imported and domestic goods. Each of these commodities, Y_i, is a CES function of inputs domestic output, Q_i, and imported goods, M_i.[6] For example, the petroleum products purchased by agents in the model are a composite of imported and domestic petroleum. By constraining all agents in the model to have the same preferences over the origin of goods we require that, for example, the agricultural and service sectors have the identical preferences over domestic oil and oil imported from the Middle East.[7] This accords with the input-output data we use and allows a very convenient nesting of production, investment and consumption decisions.

Finally, the production function includes one additional feature to allow the model to be used to examine the effects of emissions quotas or tradable permit systems: each input is used in fixed proportions to the use of an input-specific permit. The permits are owned by households and included in household wealth. Permit prices are determined endogenously by a competitive market for each type of permit. To run simulations without a permit system, the supply of permits can be set large enough so that the price of a permit goes to zero.

In each sector the capital stock changes according to the rate of fixed capital formation (J_i) and the rate of geometric depreciation (δ_i):

(2) $$\dot{K}_i = J_i - \delta_i K_i$$

Following the cost of adjustment models of Lucas (1967), Treadway (1969) and Uzawa (1969) we assume that the investment process is subject to rising marginal costs of installation. To

[6] The elasticity of substitution in this function is the Armington elasticity.

[7] This does not require that both sectors purchase the same amount of oil, or even that they purchase oil at all; only that they both feel the same way about the origins of oil they buy.

formalize this we adopt Uzawa's approach by assuming that in order to install J units of capital a firm must buy a larger quantity, I, that depends on its rate of investment (J/K):

(3) $$I_i = \left(1 + \frac{\phi_i}{2}\frac{J_i}{K_i}\right)J_i$$

where ϕ is a non-negative parameter. The difference between J and I may be interpreted various ways; we will view it as installation services provided by the capital-goods vendor

The goal of each firm is to choose its investment and inputs of labor, materials and energy to maximize intertemporal net-of-tax profits. For analytical tractability, we assume that this problem is deterministic (equivalently, the firm could be assumed to believe its estimates of future variables with subjective certainty). Thus, the firm will maximize:[8]

(4) $$\int_t^{\infty} (\pi_i - (1-\tau_4)\, P^I I_i\,)\, e^{-(R(s)-n)(s-t)}\, ds$$

where all variables are implicitly subscripted by time. The firm's profits, π, are given by:

(5) $$\pi_i = (1-\tau_2)(P_i^* Q_i - W_i L_i - P_i^E X_{iE} - P_i^M X_{iM})$$

where τ_2 is the corporate income tax, τ_4 is an investment tax credit, and P^* is the producer price of the firm's output. $R(s)$ is the long-term interest rate between periods t and s:

(6) $$R(s) = \frac{1}{s-t}\int_t^s r(v)dv$$

Because all real variables are normalized by the economy's endowment of effective labor units, profits are discounted adjusting for the rate of growth of population plus productivity growth, n.

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[8] The rate of growth of the economy's endowment of effective labor units, n, appears in the discount factor because the quantity and value variables in the model have been scaled by the number of effective labor units. These variables must be multiplied by exp(nt) to convert them back to their original form.

Solving the top tier optimization problem gives the following equations characterizing the firm's behavior:

(7)
$$X_{ij} = \delta_{ij}^{O} \left(A_i^{O}\right)^{\sigma_i^O - 1} Q_i \left(\frac{P_i^*}{P_j}\right)^{\sigma_i^O} \quad j \in \{L,E,M\}$$

(8)
$$\lambda_i = (1 + \phi_i \frac{J_i}{K_i})(1 - \tau_4)\, P^I$$

(9)
$$\frac{d\lambda_i}{ds} = (r + \delta_i)\, \lambda_i - (1 - \tau_2) P_i^* \frac{dQ_i}{dK_i} - (1 - \tau_4) P^I \frac{\phi_i}{2} \left(\frac{J_i}{K_i}\right)^2$$

where λ_i is the shadow value of an additional unit of investment in industry *i*.

Equation (7) gives the firm's factor demands for labor, energy and materials, and equations (8) and (9) describe the optimal evolution of the capital stock. By integrating (9) along the optimum path of capital accumulation, it is straightforward to show that λ_i is the increment to the value of the firm from a unit increase in its investment at time *t*. It is related to *q*, the after-tax marginal version of Tobin's Q (Abel, 1979), as follows:

(10)
$$q_i = \frac{\lambda_i}{(1 - \tau_4) P^I}$$

Thus we can rewrite (8) as:

(11)
$$\frac{J_i}{K_i} = \frac{1}{\phi_i}(q_i - 1)$$

Inserting this into (3) gives total purchases of new capital goods:

(12)
$$I_i = \frac{1}{2\phi_i}\left(q_i^2 - 1\right) K_i$$

Based on Hayashi (1979), who showed that actual investment seems to be party driven by cash flows, we modify (12) by writing I_i as a function not only of q, but also of the firm's current cash flow at time t, π_i, adjusted for the investment tax credit:

$$I_i = \alpha_2 \frac{1}{2\phi_i}\left(q_i^2 - 1\right)K_i + \left(1-\alpha_2\right)\frac{\pi_i}{\left(1-\tau_4\right)P^I} \tag{13}$$

This improves the model's ability to mimic historical data and is consistent with the existence of firms that are unable to borrow and therefore invest purely out of retained earnings. The weight on unconstrained behavior, α_2, is taken to be 0.3 based on a range of empirical estimates reported by McKibbin and Sachs (1991).

So far we have described the demand for investment goods by each sector. Investment goods are supplied, in turn, by a thirteenth industry that combines labor and the outputs of other industries to produce raw capital goods. We assume that this firm faces an optimization problem identical to those of the other twelve industries: it has a nested CES production function, uses inputs of capital, labor, energy and materials in the top tier, incurs adjustment costs when changing its capital stock, and earns zero profits. The key difference between it and the other sectors is that we use the investment column of the input-output table to estimate its production parameters.

2.2 Households

Households have three distinct activities in the model: they supply labor, they save, and they consume goods and services. Within each region we assume household behavior can be modeled by a representative agent with an intertemporal utility function of the form:

$$U_t = \int_t^\infty (\ln C(s) + \ln G(s))\, e^{-\theta(s-t)}\, ds \tag{14}$$

where $C(s)$ is the household's aggregate consumption of goods and services at time s, $G(s)$ is government consumption at s, which we take to be a measure of public goods provided, and θ is the rate of time preference.[9] The household maximizes (14) subject to the constraint that the present value of consumption be equal to the sum of human wealth, H, and initial financial assets, F:[10]

(15) $$\int_t^{\infty} P^c(s)C(s)e^{-(R(s)-n)(s-t)} = H_t + F_t$$

Human wealth is defined as the expected present value of the future stream of after-tax labor income plus transfers:

(16) $$H_t = \int_t^{\infty} (1-\tau_1)(W(L^G + L^C + L^I + \sum_{i=1}^{12} L^i) + TR)e^{-(R(s)-n)(s-t)}ds$$

where τ_1 is the tax rate on labor income, TR is the level of government transfers, L^C is the quantity of labor used directly in final consumption, L^I is labor used in producing the investment good, L^G is government employment, and L^i is employment in sector i. Financial wealth is the sum of real money balances, MON/P, real government bonds in the hand of the public, B, net holding of claims against foreign residents, A, the value of capital in each sector, and holdings of emissions permits, Q_i^P:

(17) $$F = \frac{MON}{P} + B + A + q^I K^I + q^C K^C + \sum_{i=1}^{12} q^i K^i + \sum_{i=1}^{12} P_i^P Q_i^P$$

[9] This specification imposes the restriction that household decisions on the allocations of expenditure among different goods at different points in time be separable.

[10] As before, n appears in (15) because the model's scaled variables must be converted back to their original basis.

Solving this maximization problem gives the familiar result that aggregate consumption spending is equal to a constant proportion of private wealth, where private wealth is defined as financial wealth plus human wealth:

$$P^C C = \theta(F + H) \tag{18}$$

However, based on the evidence cited by Campbell and Mankiw (1990) and Hayashi (1982) we assume some consumers are liquidity-constrained and consume a fixed fraction γ of their after-tax income (*INC*).[11] Denoting the share of consumers who are not constrained and choose consumption in accordance with (18) by α_8, total consumption expenditure is given by:

$$P^C C = \alpha_8 \theta(F_t + H_t) + (1-\alpha_8)\gamma INC \tag{19}$$

The share of households consuming a fixed fraction of their income could also be interpreted as permanent income behavior in which household expectations about income are myopic.

Once the level of overall consumption has been determined, spending is allocated among goods and services according to a two-tier CES utility function.[12] At the top tier, the demand equations for capital, labor, energy and materials can be shown to be:

$$P_i X_{Ci} = \delta_{Ci} P^C C \left(\frac{P^C}{P_i} \right)^{\sigma_C^O - 1}, i \in \{K, L, E, M\} \tag{20}$$

[11] There has been considerable debate about the empirical validity of the permanent income hypothesis. In addition the work of Campbell , Mankiw and Hayashi, other key papers include Hall (1978), and Flavin (1981). One side effect of this specification is that it prevents us from computing equivalent variation. Since the behavior of some of the households is inconsistent with (18), either because the households are at corner solutions or for some other reason, aggregate behavior is inconsistent with the expenditure function derived from our utility function.

[12] The use of the CES function has the undesirable effect of imposing unitary income elasticities, a restriction usually rejected by data. An alternative would be to replace this specification with one derived from the linear expenditure system.

where X_{Ci} is household demand for good i, σ_C^O is the top-tier elasticity of substitution and the are δ_{Ci} are the input-specific parameters of the utility function. The price index for consumption, P^C, is given by:

$$P^C = \left(\sum_{j=K,L,E,M} \delta_{Cj} P_j^{\sigma_C^O - 1} \right)^{\frac{1}{\sigma_C^O - 1}} \tag{21}$$

The demand equations and price indices for the energy and materials tiers are similar.

Household capital services consist of the service flows of consumer durables plus residential housing. The supply of household capital services is determined by consumers themselves who invest in household capital, K^C, in order to generate a desired flow of capital services, C^K, according to the following production function:

$$C^K = \alpha K^C \tag{22}$$

where α is a constant. Accumulation of household capital is subject to the condition:

$$\dot{K}^C = J^C - \delta^C K^C \tag{23}$$

We assume that changing the household capital stock is subject to adjustment costs so household spending on investment, I^C, is related to J^C by:

$$I^C = \left(1 + \frac{\phi^C}{2} \frac{J^C}{K^C} \right) J^C \tag{24}$$

Thus the household's investment decision is to choose I^C to maximize:

$$\int_t^\infty \left(P^{CK} \alpha K^c - P^I I^C \right) e^{-(R(s)-n)(s-t)} ds \tag{25}$$

where P^{CK} is the imputed rental price of household capital. This problem is nearly identical to the investment problem faced by firms, and the results are very similar. The only important difference is that no variable factors are used in producing household capital services.

2.3 The Labor Market

We assume that labor is perfectly mobile among sectors within each region but is immobile between regions. Thus, wages will be equal across sectors within each region, but will generally not be equal between regions. In the long run, labor supply is completely inelastic and is determined by the exogenous rate of population growth. Long run wages adjust to move each region to full employment. In the short run, however, nominal wages are assumed to adjust slowly according to an overlapping contracts model where wages are set based on current and expected inflation and on labor demand relative to labor supply. This can lead to short-run unemployment if unexpected shocks cause the real wage to be too high to clear the labor market. At the same time, employment can temporarily exceed its long run level if unexpected events cause the real wage to be below its long run equilibrium.

2.4 Government

We take each region's real government spending on goods and services to be exogenous and assume that it is allocated among inputs in fixed proportions, which we set to 1987 values. Total government outlays include purchases of goods and services plus interest payments on government debt, investment tax credits and transfers to households. Government revenue comes from sales taxes, corporate and personal income taxes, and from sales of new government bonds. In addition, there can be taxes on externalities such as carbon dioxide emissions. The government budget constraint may be written in terms of the accumulation of public debt as follows:

(26) $$\dot{B}_t = D_t = r_t B_t + G_t + TR_t - T_t$$

where B is the stock of debt, D is the budget deficit, G is total government spending on goods and services, TR is transfer payments to households, and T is total tax revenue net of any investment tax credit.

We assume that agents will not hold government bonds unless they expect the bonds to be paid off eventually and accordingly impose the following transversality condition:

(27) $$\lim_{s\to\infty} B(s)\, e^{-(R(s)-n)s} = 0$$

This prevents per capita government debt from growing faster than the interest rate forever. If the government is fully leveraged at all times, (27) allows (26) to be integrated to give:

(28) $$B_t = \int_t^{\infty} (T - G - TR) e^{-(R(s)-n)(s-t)} ds$$

Thus, the current level of debt will always be exactly equal to the present value of future budget surpluses.[13]

The implication of (28) is that a government running a budget deficit today must run an appropriate budget surplus as some point in the future. Otherwise, the government would be unable to pay interest on the debt and agents would not be willing to hold it. To ensure that (28) holds at all points in time we assume that the government levies a lump sum tax in each period equal to the value of interest payments on the outstanding debt.[14] In effect, therefore, any increase in government debt is financed by consols, and future taxes are raised enough to accommodate the increased interest costs. Other fiscal closure rules are possible, such as requiring the ratio of

[13] Strictly speaking, public debt must be less than or equal to the present value of future budget surpluses. For tractability we assume that the government is initially fully leveraged so that this constraint holds with equality.

[14] In the model the tax is actually levied on the difference between interest payments on the debt and what interest payments would have been if the debt had remained at its base case level. The remainder, interest payments on the base case debt, is financed by

government debt to GDP to be unchanged in the long run. These closures have interesting implications but are beyond the scope of this paper.

2.5 Financial Markets and the Balance of Payments

The eight regions in the model are linked by flows of goods and assets. Flows of goods are determined by the import demands described above. These demands can be summarized in a set of bilateral trade matrices which give the flows of each good between exporting and importing countries. There is one eight by eight trade matrix for each of the twelve goods.

Trade imbalances are financed by flows of assets between countries. Each region with a current account deficit will have a matching capital account surplus, and vice versa.[15] We assume asset markets are perfectly integrated across regions.[16] With free mobility of capital, expected returns on loans denominated in the currencies of the various regions must be equalized period to period according to a set of interest arbitrage relations of the following form:

$$i_k + \mu_k = i_j + \mu_j + \frac{\dot{E}_k^j}{E_k^j} \tag{29}$$

where i_k and i_j are the interest rates in countries k and j, μ_k and μ_j are exogenous risk premiums demanded by investors (possibly zero), and E_k^j is the exchange rate between the currencies of the two countries.[17]

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ordinary taxes.

[15] Global net flows of private capital are constrained to be zero at all times – the total of all funds borrowed exactly equals the total funds lent. As a theoretical matter this may seem obvious, but it is often violated in international financial data.

[16] The mobility of international capital is a subject of considerable debate; see Gordon and Bovenberg (1994) or Feldstein and Horioka (1980).

[17] The one exception to this is the oil exporting region, which we treat as choosing its foreign lending in order to maintain a desired ratio of income to wealth.

Capital flows may take the form of portfolio investment or direct investment but we assume these are perfectly substitutable *ex ante*, adjusting to the expected rates of return across economies and across sectors. Within each economy, the expected returns to each type of asset are equated by arbitrage, taking into account the costs of adjusting physical capital stock and allowing for exogenous risk premiums. However, because physical capital is costly to adjust, any inflow of financial capital that is invested in physical capital will also be costly to shift once it is in place. This means that unexpected events can cause windfall gains and losses to owners of physical capital and *ex post* returns can vary substantially across countries and sectors. For example, if a shock lowers profits in a particular industry, the physical capital stock in the sector will initially be unchanged but its financial value will drop immediately.

2.6 Money Demand

Finally, we assume that money enters the model via a constraint on transactions.[18] We use a money demand function in which the demand for real money balances is a function of the value of aggregate output and short-term nominal interest rates:

(30) $$MON = PY\, i^{\varepsilon}$$

where Y is aggregate output, P is a price index for Y, i is the interest rate, and ε is the interest elasticity of money demand. Following McKibbin and Sachs (1991) we take ε to be -0.6. The supply of money is determined by the balance sheet of the central bank and is exogenous.

[18] Unlike other components of the model we simply assume this rather than deriving it from optimizing behavior. Money demand can be derived from optimization under various assumptions: money gives direct utility; it is a factor of production; or it must be used to conduct transactions. The distinctions are unimportant for our purposes.

2.7 Parameterization

To estimate G-Cubed's parameters we began by constructing a consistent time series of input-output tables for the United States. The procedure is described in detail in McKibbin and Wilcoxen (1994) and can be summarized as follows. We started with the detailed benchmark U.S. input-output transactions tables produced by the Bureau of Economic Analysis (BEA) for years 1958, 1963, 1967, 1972, 1977 and 1982.[19] Our first step was to convert these to a standard set of industrial classifications and then aggregate them to twelve sectors.[20] Second, we corrected the treatment of consumer durables, which are included in consumption rather than investment in the U.S. National Income and Product Accounts and the benchmark input-output tables. Third, we supplemented the value added rows of the tables using a detailed dataset on capital and labor input by industry constructed by Dale Jorgenson and his colleagues.[21] Finally, we obtained prices for each good in each benchmark year from the output and employment data set constructed by the Office of Employment Projections at the Bureau of Labor Statistics (BLS). Table 3 shows the relationship between G-Cubed sectors and the Standard Industrial Classification, the industry definitions used by the BEA and the BLS.

This dataset allowed us to estimate the model's parameters for the United States. To estimate the production side of the model, we began with the energy and materials tiers because

[19] A benchmark table exists for 1947 also exists but has inadequate final demand detail for our purposes. Also, a table for 1987 has recently become available.

[20] Converting the data to a standard basis was necessary because the sector definitions and accounting conventions used by the BEA have changed over time.

[21] Primary factors often account for half or more of industry costs so it is particularly important that this part of the data set be constructed as carefully as possible. From the standpoint of estimating cost and production functions, however, value added is the least satisfactory part of the benchmark input-output tables. In the early tables, labor and capital are not disaggregated. In all years, the techniques used by the BEA to construct implicit price deflators for labor and capital are subject to various methodological problems. One example is that the income of proprietors is not split between capital and imputed labor income correctly. The Jorgenson dataset corrects these problems and is the work of several people over many years. In addition to Dale Jorgenson, some of the contributors were L. Christensen, Barbara Fraumeni, Mun Sing Ho and Dae Keun Park. The original source of the data is the

they have constant returns to scale and all inputs are variable. In this case it is convenient to replace the production function with its dual unit cost function. For industry *i*, the unit cost function for energy is:

$$c_i^E = \frac{1}{A_i^E}\left(\sum_{k=1}^{5} \delta_{ik}^E \, p_{ik}^{1-\sigma_i^E}\right)^{\frac{1}{1-\sigma_i^E}} \tag{31}$$

The cost function for materials has a similar form. Assuming that the energy and materials nodes earn zero profits, *c* will be equal to the price of the node's output. Using Shepard's Lemma to derive demand equations for individual commodities and then converting these demands to cost shares gives expressions of the form:

$$s_{ij}^E = \delta_{ij}^E \left(\frac{P_j}{A_i^E P_i}\right)^{1-\sigma_i^E}, \; j = 1,\ldots,5 \tag{32}$$

where s_{ij}^E is the share of industry *i*'s spending on energy that is devoted to purchasing input *j*.[22] A_i^E, σ_i^E, and δ_{ij}^E were found by estimating (31) and (32) as a system of equations.[23] Estimates of the parameters in the materials tier were found by an analogous approach.

The output node must be treated differently because it includes capital, which is not variable in the short run. We assume that the firm chooses output, Q_i, and its top-tier variable inputs (*L*, *E* and *M*) to maximize its restricted profit function, π:

$$\pi_i = p_i Q_i - \sum_{j=L,E,M} p_j X_{ij} \tag{33}$$

Fourteen Components of Income tape produced by the Bureau of Economic Analysis. See Ho (1989) for more information.

[22] When σ^E is unity, this collapses to the familiar Cobb-Douglas result that s=δ and is independent of prices.

[23] For factors for which the value of *s* was consistently very small, we set the corresponding input to zero and estimated the production function over the remaining inputs.

where the summation is taken over all inputs other than capital. Inserting the production function into (33) and rewriting gives:

$$(34) \qquad \pi_i = P_i A_i^O \left(\delta_{ik}^{\frac{1}{\sigma_i^O}} K_i^{\frac{\sigma_i^O - 1}{\sigma_i^O}} + \sum_{j=L,E,M} \delta_{ij}^{\frac{1}{\sigma_i^O}} X_{ij}^{\frac{\sigma_i^O - 1}{\sigma_i^O}} \right)^{\frac{\sigma_i^O}{\sigma_i^O - 1}} - \sum_{j=L,E,M} P_j X_{ij}$$

where K_i is the quantity of capital owned by the firm, δ_{ik} is the distributional parameter associated with capital, and j ranges over inputs other than capital. Maximizing (34) with respect to variable inputs produces the following factor demand equations for industry i:

$$(35) \qquad X_{ij} = \delta_{ij} P_j^{-\sigma_i^O} \delta_{ik}^{\frac{1}{\sigma_i^O - 1}} K_i \left(\left(P_i A_i^O\right)^{1-\sigma_i^O} - \sum_k \delta_{ik} P_k^{1-\sigma_i^O} \right)^{\frac{\sigma_i^O}{1-\sigma_i^O}}, \forall j \in \{L, E, M\}$$

This system of equations can be used to estimate the top-tier production parameters.

The parameter estimates for all three tiers, along with standard errors, are shown in Table 4.[24] Parameters shown without standard errors were imposed. Most of these are δ parameters for inputs which were negligible. A few elasticities were imposed because their estimated values came out negative or because it was impossible to get the estimation procedure to converge otherwise. One broad conclusion from Table 4 is that most of the nodes have elasticities fairly far from unity. For the output elasticities, in particular, statistical tests would strongly reject the hypothesis that the output node is Cobb-Douglas.

Much of the empirical literature on cost and production functions fails to account for the fact that capital is fixed in the short run. Rather than using (35), a common approach is to use factor demands of the form:

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[24] Standard errors are shown rather than *t*-statistics because for most of the parameters zero is not an interesting or relevant null hypothesis. Virtually all of the parameters are significantly different from zero, as would be expected.

$$(36) \qquad X_{ij} = \delta_{ij}\, P_i^{-\sigma_i^O}\, \frac{Q_i}{A_i^O} \left(\sum_{k=K,L,E,M} \delta_{ik} P_k^{1-\sigma_i^O} \right)^{\frac{\sigma_i^O}{1-\sigma_i^O}}$$

As shown above, this expression is correct only if all inputs are variable in the short run. Equation (36) differs from (35) in one very important respect: (36) has constant returns to scale. This implies that Q is exogenous in (36), both in terms of economic interpretation and econometric specification. In other words, the firm's supply curve will be perfectly horizontal and the firm will be indifferent about its scale of output. In (35), however, Q is implicitly endogenous while the price of output, P, is exogenous. As a result, the firms described by (35) have upward-sloping marginal cost curves.

Using (36) instead of (35) may bias the estimated elasticity of substitution downward in a relatively inflexible econometric specification such as the CES. The fixed nature of the capital stock would be reflected in the parameter estimates as a lack of substitutability among inputs. To gauge the empirical significance of using the correct specification (35) rather than (36) we estimated the output node using both specifications. The result for the variable-capital case are shown in Table 5.

Comparing Table 4 and Table 5 shows that the treatment of capital has a very significant effect on the estimated elasticities of substitution. The estimates in Table 5 are not all biased downward but virtually all are biased toward unity, some substantially so. In oil refining, for example, the fixed-capital estimate for the top tier elasticity, $\sigma_3{}^O$, is 0.54 while in the variable elasticity case it is 1.04. Considering the nature of the industry, the lower estimate from the fixed-capital case seems more reasonable than an elasticity of unity. The results for other sectors are similarly intuitive. We conclude from this that it is essential to account for capital correctly in order to obtain useful estimates of elasticities of substitution.

The main limitation of our approach to parameterizing the model is that our data set contains few observations because there are very few benchmark input-output tables. In future work we plan to extend the data set back to include the 1947 input-output table and forward to include benchmark tables built by the Bureau of Labor Statistics for 1987 and 1990. This would improve the parameter estimates by increasing the number of data points substantially.

Estimating parameters for regions other than the United States is more difficult because even less time-series input-output data is available. In part this is because some countries do not collect the data regularly and in part it is because half of G-Cubed's geographic entities are regions rather than individual countries. As a result, we impose the restriction that substitution elasticities within individual industries are equal across regions.[25] By doing so, we are able to use the U.S. elasticity estimates everywhere. The share parameters (the δ's in the equations above), however, are derived from regional input-output data and generally differ from one region to another. The share parameters for the United States are taken from a 1987 U.S. input-output table prepared by the Bureau of Labor Statistics. Those for Japan, Australia, China and the Former Soviet Union have been taken from input-output tables for each region. The share parameters for other regions are calculated by adjusting U.S. share parameters to account for actual final demand data from the national accounts for the corresponding region. In effect, we are assuming that all regions share a similar but not identical production technology. This is intermediate between one extreme of assuming that the regions share common technologies and the other extreme of allowing the technologies to differ in arbitrary ways. The regions also differ in their endowments of primary factors, their government policies, and patterns of final demands.

[25] For example, the top tier elasticity of substitution is identical in the durable goods industries of Japan and the United States. This approach is consistent with the econometric evidence of Kim and Lau (1994). This specification does *not* mean, however, that the elasticities are the same across industries *within* a country.

Final demand parameters, such as those in the utility function or in the production function of new investment goods were estimated by a similar procedure: elasticities were estimated from U.S. data and share parameters were obtained from regional input-output tables. Trade shares were obtained from 1987 United Nations Standard Industry Trade Classification (SITC) data aggregated up from the four-digit level.[26] We did not have access to adequate time-series data to estimate the trade elasticities for all regions, however, so in the current version of the model they have been imposed to be unity.

2.8 Solution Algorithm

Although G-Cubed does not have an especially large number of sectors, or even an especially large number of regions, it is still quite challenging to solve because it includes foresight and over 100 forward-looking costate variables. It is solved using an algorithm and computer software developed by McKibbin (1986).[27] To describe the solution procedure we begin by observing that from a mathematical standpoint, G-Cubed is a system of simultaneous equations which can be written in the form:

$$Z_t = F(Z_t, S_t, C_t, X_t) \tag{37}$$

$$S_{t+1} - S_t = G(Z_t, S_t, C_t, X_t) \tag{38}$$

$$C_{t+1} - C_t = H(Z_t, S_t, C_t, X_t) \tag{39}$$

where Z is a vector of endogenous variables, S is a vector of state variables, C is a vector of costate variables, X is a vector of exogenous variables, and F, G and H are vector functions. The first step in constructing the solution is to use numerical differentiation to linearize (37)-(39)

[26] A full mapping of SITC codes into G-Cubed industries is contained in McKibbin and Wilcoxen (1994).

[27] For a more detailed description of the algorithm, see McKibbin and Sachs (1991), Appendix C.

around the model's database. We then transform the model into its minimal state space representation by using (37) to find a set of equations that allow us to eliminate Z from (38) and (39):

(40) $$S_{t+1} - S_t = G(f(S_t, C_t, X_t), S_t, C_t, X_t)$$

(41) $$C_{t+1} - C_t = H(f(S_t, C_t, X_t), S_t, C_t, X_t)$$

The linearized model is then in the form:

(42) $$dS_{t+1} = (I + G_Z f_S + G_S)\,dS_t + (G_Z f_C + G_C)\,dC_t + (G_Z f_X + G_X)\,dX_t$$

(43) $$dC_{t+1} = (I + H_Z f_C + H_C)\,dC_t + (H_Z f_S + H_S)\,dS_t + (H_Z f_X + H_X)\,dX_t$$

The eigenvalues of this system of equations are then calculated to ensure that the condition for saddle-point stability is satisfied (that is, that the number of eigenvalues outside the unit circle are equal to the number of costate variables). Following that we compute the model's stable manifold as follows. For convenience, define Γ:

(44) $$\Gamma = (I + H_Z f_C + H_C)^{-1}$$

Using Γ we can rewrite (43) to give dC_t in terms of the other variables:

(45) $$dC_t = \Gamma dC_{t+1} - \Gamma(H_Z f_S + H_S)dS_t - \Gamma(H_Z f_X + H_X)dX_t$$

Substituting (45) into (42) gives:

(46) $$\begin{aligned} dS_{t+1} = {} & (I + G_Z f_S + G_S - (G_Z f_C + G_C)\Gamma(H_Z f_S + H_S))\,dS_t \\ & + (G_Z f_C + G_C)\,\Gamma\, dC_{t+1} \\ & + (G_Z f_X + G_X - (G_Z f_C + G_C)\Gamma(H_Z f_X + H_X))\,dX_t \end{aligned}$$

Applying (45) recursively and using (46) allows us to find an expression for the stable manifold for the costate variables in terms of changes in current state variables and all current and future changes in the exogenous variables. The expression will have the following form:

(47) $$dC_t = \Phi\, dS_t + \sum_{i=t}^{T} \Theta_i\, dX_i + \Omega\, dC_T$$

where Φ, Θ_i, and Ω are large matrices of constants. We evaluate Φ, Θ, and Ω numerically; in general, their closed-form expressions will be quite complicated. Once these matrices have been calculated, the model can be solved quickly and easily for different experiments because the new values of the costate variables can be obtained simply by evaluating (47). These values can then be inserted into (37) to calculate the values of other endogenous variables.

3 Generating a Baseline

Because G-Cubed is an intertemporal model, it is necessary to calculate a baseline, or "business as usual", solution before the model can be used for policy simulations. In order to do so we begin by making assumptions about the future course of key exogenous variables. We take the underlying long-run rate of world population growth plus productivity growth to be 2.5 percent per annum, and take the long-run real interest rate to be 5 percent. We also assume that tax rates and the shares of government spending devoted to each commodity remain unchanged. Our remaining assumptions are listed by region in Table 6. Some of these are not much more than rough guesses; the model would benefit from further refinement in this area.

Because these assumptions do not necessarily match the expectations held by agents in the real world, the model's solution in any given year, say 1990, will generally not reproduce that year's historical data exactly. In particular, it is unlikely that the costate variables based on current and expected future paths of the exogenous variables in the *model* will equal the *actual* values of those variables in 1990. This problem arises in all intertemporal models and is not unique to G-Cubed but it is inconvenient when interpreting the model's results.

To address the problem we add a set of constants, one for each costate variable, to the model's costate equations. For example, the constants for Tobin's *q* for each sector in each country are added to the arbitrage equation for each sector's *q*. Similarly, constants for each real exchange rate are added to the interest arbitrage equation for each country, and a constant for human wealth is added to the equation for human wealth.[28] To calculate the constants we use Newton's Method to find a set of values that will make the model's costate variables in 1990 exactly equal their 1990 historical values. After the constants have been determined, the model will reproduce the base year exactly given the state variables inherited from 1989 and the assumed future paths of all exogenous variables.[29]

One additional problem is to solve for both real and nominal interest rates consistently since the real interest rate is the nominal interest rate from the money market equilibrium less the *ex ante* expected inflation rate. To produce the expected inflation rate implicit in historical data for 1990 we add a constant to the equation for nominal wages in each country.[30]

Finally, we are then able to construct the baseline trajectory by solving the model for each period after 1990 given any shocks to variables, shocks to information sets (announcements about future policies), or changes in initial conditions.

4 Conclusion

G-Cubed is a detailed, comprehensive, world economic model suitable for analyzing the effects of a wide range of policies on international trade and financial flows. Its key features

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[28] One interpretation of these constants is that they are risk premiums; another is that they are simply the residuals left between the actual data and the econometrically fitted values calculated by the model.

[29] In general, these constants affect the model's steady state but have little or no effect on the transitional dynamics.

[30] One way to interpret this is as a shift in the full employment level of unemployment. In that case this approach is equivalent to using the full model to solve for the natural rate of unemployment in each country.

include: eight geographic regions; twelve sectors in each region; international trade modeled at the bilateral level; fully endogenous international capital flows; intertemporal optimization used to model saving, investment, and international asset market arbitrage; where appropriate, the existence of liquidity-constrained agents is taken into account; behavioral parameters estimated from time-series data wherever possible; and all budget constraints are satisfied at all times.

G-Cubed could, however, be improved in several areas. First, the current nested CES utility function implies that budget shares will be independent of income, a fact which is clearly inconsistent with empirical studies. This could be addressed by moving to the Linear Expenditure System or another specification that does not impose homotheticity.

A second caveat is that G-Cubed's parameter estimates for several countries, particularly those outside the OECD, are derived from time-series estimates from U.S. data. This is an unfortunate necessity brought on by the lack of time series input-output data for many developing countries. Additional data would substantially improve G-Cubed's representation of non-OECD production. A related point is that G-Cubed does not include any special treatment of the informal sector in developing countries.

Despite these caveats, G-Cubed provides a rigorous, empirically-based tool for studying economic policy in an international context. It has been used to examine carbon dioxide abatement, energy policy, tax policy, monetary and fiscal policy, the Uruguay Round of the GATT, other regional trading arrangements, financial and economic reform in developing countries, and other issues related to productivity growth and trade. These studies have consistently shown that G-Cubed's key innovations – intertemporal optimization in savings and investment combined econometric parameter estimates and full intertemporal modeling of international trade and asset flows – contribute substantially to the analysis of policies with intertemporal or international effects.

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Table 1: The Main Features of G-Cubed

- Disaggregated into eight geographic regions
- Each region's production, consumption and international trade is disaggregated into twelve sectors;
- Complete specification of the demand and supply sides of each economy;
- Full integration of real and financial markets;
- Complete intertemporal accounting linking stocks and flows of both real and financial assets;
- Imposition of al intertemporal budget constraints on agents and countries;
- Short run behavior is a weighted average of neoclassical optimization and liquidity-constrained behavior;
- Full short and long run macroeconomic closure around a long run Solow/Swan neoclassical growth model;
- Solved at an annual frequency for a full rational expectation equilibrium out to 2050 or beyond.

Table 2: Regions and Sectors in G-Cubed

Regions
1. United States
2. Japan
3. Australia
4. Rest of the OECD
5. Eastern Europe and the Former Soviet Union
6. China
7. Oil exporting developing countries
8. Other developing countries
Sectors
1. Electric utilities
2. Gas utilities
3. Petroleum refining
4. Coal mining
5. Crude oil and gas extraction
6. Other mining
7. Agriculture
8. Forestry and wood products
9. Durable goods
10. Nondurables
11. Transportation
12. Services

Table 3: G-Cubed Sectors in terms of BEA, SIC and BLS classifications

	G-Cubed Industry Name	BEA Codes	SIC Codes	BLS Codes
1	Electric utilities	68.01, 78.02, 79.02	491, B	155, 214, 219
2	Gas utilities	68.02	492, B	156
3	Petroleum refining	31	29	138, 139
4	Coal mining	7	12	7
5	Crude oil and gas extraction	8	13	8, 9
6	Mining	5, 6, 9, 10	10, 14	6, 10
7	Agriculture, fishing and hunting	1, 2, 4	01, 02, 07, 09	1-3, 5
8	Forestry and wood products	3, 20, 21	04	4, 30-36
9	Durable manufacturing	13, 22, 23, 35-64	24, 25, 32-39	37-103
10	Nondurable manufacturing	14-19, 24-30, 32-34	20-23, 26-28, 30, 31	104-137, 140-144
11	Transportation	65	40-42, 44-47, A	145-152, 218
12	Services	66, 67, 69, 70-77, 68.03, 78.01, 78.03, 78.04, 79.01, 79.03	48, 494-497, 50-65, 67, 70, 72, 73, 75, 76, 78-84, 86-89, C	153,154,157-166, 168-211, 213,216,220

A) Includes local government transit, for which no SIC code exists.
B) Includes part of SIC 493 (Combined Services).
C) Includes government enterprises other than local transit and electric utilities.

Table 4: Estimation Results, Fixed Capital Stock

	1. Electric Utilities	2. Gas Utilities	3. Oil Refining	4. Coal Mining
Energy Tier				
σ^E	0.2	0.9325 (0.3473)	0.2	0.1594 (0.1208)
A^E	1.0978 (0.0225)	1.2968 (1.6823)	0.9997 (0.0016)	1.0290 (0.0042)
δ_1	0.	0.	0.0060 (0.0009)	0.1028 (0.0062)
δ_2	0.3300 (0.0179)	0.6426 (0.0309)	0.0175 (0.0012)	0.
δ_3	0.2356 (0.0475)	0.	0.1045 (0.0033)	0.1007 (0.0190)
δ_4	0.4344 (0.0318)	0.	0.	0.7965 (0.0238)
δ_5	0.	0.3574 (0.0309)	0.8720 (0.0050)	0.
Materials Tier				
σ^M	1.	0.2	0.2	0.5294 (0.0187)
A^M	1.	1.0748 (0.0205)	1.0535 (0.0050)	1.0258 (0.0037)
δ_6	0.	0.	0.	0.
δ_7	0.	0.	0.	0.
δ_8	0.	0.	0.	0.0240 (0.0033)
δ_9	0.1539 (0.0209)	0.0999 (0.0163)	0.0863 (0.0109)	0.4034 (0.0152)
δ_{10}	0.0500 (0.0034)	0.0424 (0.0033)	0.2168 (0.0142)	0.1157 (0.0055)
δ_{11}	0.3627 (0.0125)	0.0807 (0.0265)	0.3125 (0.0143)	0.0437 (0.0056)
δ_{12}	0.4334 (0.0116)	0.7769 (0.0302)	0.3844 (0.0190)	0.4133 (0.0146)
Output Tier				
σ^O	0.7634 (0.0765)	0.8096 (0.0393)	0.5426 (0.0392)	1.7030 (0.0380)
A^O	1.1188 (0.0363)	1.2626 (0.0072)	0.9791 (0.0020)	1.3681 (0.0638)
δ_K	0.3851 (0.0277)	0.2466 (0.0053)	0.0736 (0.0025)	0.3669 (0.0242)
δ_L	0.2150 (0.0137)	0.1332 (0.0095)	0.0555 (0.0047)	0.3058 (0.0142)
δ_E	0.2585 (0.0364)	0.5799 (0.0118)	0.7592 (0.0102)	0.1088 (0.0093)
δ_M	0.1413 (0.0051)	0.0403 (0.0030)	0.1118 (0.0034)	0.2185 (0.0035)

Table 4, continued: Estimation Results, Fixed Capital Stock

	5. Crude Oil, Gas	6. Other Mining	7. Agriculture	8. Forestry
Energy Tier				
σ^E	0.1372 (0.0339)	1.1474 (0.1355)	0.6277 (0.0510)	0.9385 (0.1380)
A^E	0.9920 (0.0159)	0.9401 (0.0685)	1.0208 (0.0157)	1.2990 (0.7145)
δ_1	0.1137 (0.0149)	0.5129 (0.0185)	0.1488 (0.0204)	0.3489 (0.0228)
δ_2	0.0448 (0.0069)	0.1727 (0.0139)	0.0258 (0.0066)	0.0993 (0.0097)
δ_3	0.1077 (0.0116)	0.2891 (0.0195)	0.8254 (0.0267)	0.5377 (0.0233)
δ_4	0.	0.0253 (0.0043)	0.	0.0141 (0.0045)
δ_5	0.7337 (0.0286)	0.	0.	0.
Materials Tier				
σ^M	0.2	2.7654 (0.0278)	1.7323 (0.1052)	0.1757 (0.0000)
A^M	1.0442 (0.0079)	0.9815 (0.0035)	0.9924 (0.0072)	1.0046 (0.0025)
δ_6	0.	0.1510 (0.0121)	0.	0.
δ_7	0.	0.	0.5350 (0.0178)	0.0583 (0.0043)
δ_8	0.	0.	0.	0.5934 (0.0117)
δ_9	0.1461 (0.0212)	0.2946 (0.0143)	0.0225 (0.0015)	0.0792 (0.0112)
δ_{10}	0.0417 (0.0037)	0.1318 (0.0065)	0.1997 (0.0125)	0.0594 (0.0033)
δ_{11}	0.0353 (0.0075)	0.0570 (0.0105)	0.0278 (0.0016)	0.0615 (0.0058)
δ_{12}	0.7769 (0.0243)	0.3656 (0.0226)	0.2151 (0.0054)	0.1483 (0.0069)
Output Tier				
σ^O	0.4934 (0.0310)	1.0014 (0.3146)	1.2830 (0.0469)	0.9349 (0.0802)
A^O	1.7834 (0.0785)	0.0001 (0.0009)	0.8650 (0.0051)	0.9741 (0.0107)
δ_K	0.5849 (0.0095)	0.2302 (0.8571)	0.1382 (0.0101)	0.1140 (0.0130)
δ_L	0.1670 (0.0068)	0.3214 (0.3698)	0.2471 (0.0113)	0.2747 (0.0087)
δ_E	0.0497 (0.0069)	0.0698 (0.0896)	0.0194 (0.0020)	0.0251 (0.0033)
δ_M	0.1984 (0.0049)	0.3786 (0.3979)	0.5953 (0.0022)	0.5862 (0.0087)

Table 4, continued: Estimation Results, Fixed Capital Stock

	9. Durables	10. Nondurables	11. Transp.	12. Services
Energy Tier				
σ^E	0.8045 (0.0582)	1.	0.2	0.3211 (0.0449)
A^E	3.8779 (1.5069)	1.	1.0379 (0.0054)	1.0086 (0.0052)
δ_1	0.5019 (0.0251)	0.3492 (0.0105)	0.0581 (0.0060)	0.4313 (0.0062)
δ_2	0.	0.2374 (0.0124)	0.	0.1619 (0.0055)
δ_3	0.3013 (0.0070)	0.2962 (0.0145)	0.9419 (0.0060)	0.4068 (0.0047)
δ_4	0.1968 (0.0236)	0.0304 (0.0025)	0.	0.
δ_5	0.	0.0868 (0.0078)	0.	0.
Materials Tier				
σ^M	0.2	0.0573 (0.0000)	0.2	3.0056 (0.0728)
A^M	1.0287 (0.0033)	1.0412 (0.0034)	1.1182 (0.0117)	0.9867 (0.0008)
δ_6	0.0265 (0.0032)	0.	0.	0.
δ_7	0.	0.1841 (0.0095)	0.	0.
δ_8	0.	0.	0.	0.
δ_9	0.6592 (0.0115)	0.0591 (0.0020)	0.1300 (0.0032)	0.0938 (0.0046)
δ_{10}	0.0913 (0.0036)	0.5263 (0.0053)	0.0550 (0.0041)	0.1349 (0.0106)
δ_{11}	0.0436 (0.0015)	0.0487 (0.0018)	0.3673 (0.0274)	0.0347 (0.0012)
δ_{12}	0.1794 (0.0114)	0.1817 (0.0047)	0.4477 (0.0219)	0.7366 (0.0128)
Output Tier				
σ^O	0.4104 (0.0193)	1.0044 (0.0117)	0.5368 (0.0700)	0.2556 (0.0272)
A^O	1.0124 (0.0029)	0.9496 (0.0057)	0.9236 (0.0138)	1.0000 (0.0164)
δ_K	0.0682 (0.0011)	0.1034 (0.0038)	0.1263 (0.0082)	0.1942 (0.0033)
δ_L	0.3402 (0.0027)	0.2613 (0.0027)	0.4876 (0.0055)	0.4764 (0.0129)
δ_E	0.0312 (0.0016)	0.0167 (0.0016)	0.0776 (0.0089)	0.0312 (0.0008)
δ_M	0.5604 (0.0018)	0.6186 (0.0015)	0.3086 (0.0054)	0.2982 (0.0109)

Table 5: Estimation Results, Variable Capital Stock

	1. Electric Utilities	2. Gas Utilities	3. Oil Refining
σ^O	0.8662 (0.0100)	0.7812 (0.0010)	1.0381 (0.0089)
A^O	1.2518 (0.0546)	1.2353 (0.0339)	1.0884 (0.0308)
δ_K	0.4777 (0.0112)	0.2355 (0.0066)	0.1177 (0.0049)
δ_L	0.2604 (0.0046)	0.1368 (0.0087)	0.1382 (0.0046)
δ_E	0.1399 (0.0081)	0.5867 (0.0178)	0.5533 (0.0116)
δ_M	0.1220 (0.0040)	0.0411 (0.0041)	0.1908 (0.0040)
	4. Coal Mining	**5. Crude Oil & Gas**	**6. Mining**
σ^O	0.9903 (0.0006)	0.9537 (0.0074)	1.0014 (0.0006)
A^O	1.3910 (0.1009)	2.1141 (0.2181)	0.0001 (0.0000)
δ_K	0.1939 (0.0066)	0.4779 (0.0046)	0.2305 (0.0148)
δ_L	0.3947 (0.0079)	0.1891 (0.0078)	0.3213 (0.0089)
δ_E	0.1612 (0.0039)	0.0426 (0.0025)	0.0698 (0.0131)
δ_M	0.2501 (0.0037)	0.2904 (0.0126)	0.3784 (0.0054)
	7. Agriculture	**8. Forestry**	**9. Durables**
σ^O	1.1503 (0.0140)	0.9465 (0.0196)	1.0432 (0.0108)
A^O	0.8864 (0.0096)	0.9643 (0.0160)	0.9856 (0.0188)
δ_K	0.1738 (0.0037)	0.1243 (0.0081)	0.1133 (0.0028)
δ_L	0.2106 (0.0057)	0.2898 (0.0074)	0.3240 (0.0031)
δ_E	0.0254 (0.0021)	0.0179 (0.0012)	0.0184 (0.0011)
δ_M	0.5902 (0.0022)	0.5679 (0.0045)	0.5444 (0.0032)
	10. Nondurables	**11. Transportation**	**12. Services**
σ^O	0.9832 (0.0001)	0.8602 (0.0136)	0.9428 (0.0043)
A^O	0.9362 (0.0199)	0.9353 (0.0366)	0.9954 (0.0253)
δ_K	0.1019 (0.0018)	0.1622 (0.0048)	0.2217 (0.0080)
δ_L	0.2620 (0.0025)	0.4737 (0.0063)	0.4789 (0.0047)
δ_E	0.0154 (0.0008)	0.0553 (0.0035)	0.0219 (0.0014)
δ_M	0.6208 (0.0024)	0.3088 (0.0091)	0.2776 (0.0092)

Table 6: Regional Assumptions Used in Generating the Baseline

	USA	Japan	Australia	Other OECD	China	Other Developing Countries	Eastern Europe and Former Soviet Union
Population growth	0.5%	0.0%	0.8%	0.7%	1.5%	1.0%	0.5%
Non-energy productivity growth	2.0%	2.5%	2.2%	2.3%	4%	2.5%	2.0%
Energy sector productivity growth	1.5%	2.0%	1.7%	1.8%	4%	2.5%	1.5%
Energy efficiency growth	1%	1%	1%	1%	1%	1%	1%
Monetary policy (fixed money growth rate)	2.9 %	1.25 %	1.64 %	3.98 %	12.84%	6.48%	23.81%